The PytEuk Table

A mathematical exhibit around triangles

Contents

1. Aim

Four results about right triangles can be discovered with this exhibit:

1. Pythagoras' Theorem: The square of the hypotenuse equals the sum of the squares of the two legs.
2. The square of a leg equals the product of the hypotenuse and the projection of the leg on the hypotenuse.
3. The square of the altitude (the altitude towards the hypotenuse) equals the product of the projections of the two legs on the hypotenuse.
4. The altitude towards the hypotenuse cuts a right triangle into two right triangles which are similar to it.

2. General Overview and Comments

With the help of three puzzles one can see that certain surfaces have the same area, and then discover the first three of the above theorems. The last result can be checked with the two given triangular pieces.

The puzzle pieces are all rectangular, plus there are two special triangular pieces.

There are two large holes to contain the pieces. The holes should be less deep than the pieces, and possibly (just minimally) slightly larger than what depicted to allow all pieces to enter smoothly.

There is an explanation panel (possibly in various languages).

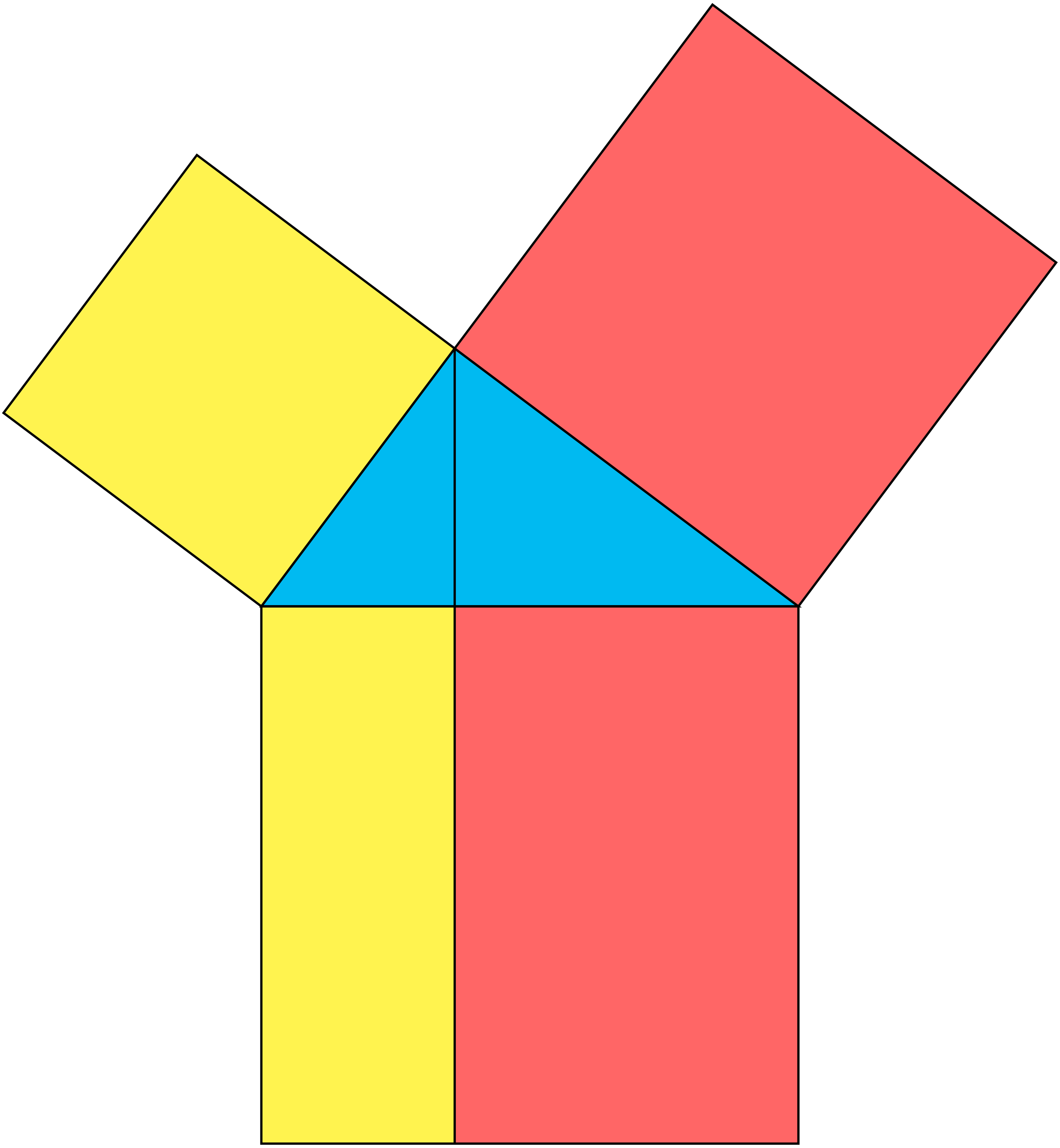
Finally, all is contained in a square table (with removable legs, so that it can be easily transported).

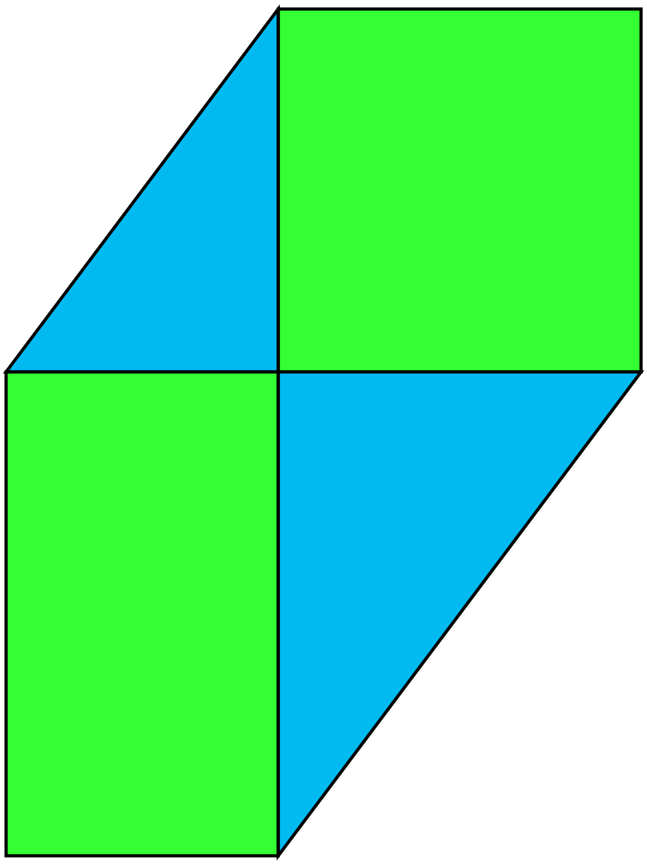
The best material for the puzzle pieces is probably lacquered wood.

Notice that although several colors are used the activity is also understandable for color blind people.

The idea for this exhibit originated from an exhibit from the Mathematikum Museum (Giessen, Germany), where Pythagoras' Theorem can be discovered – in a particular example – by arranging 25 square pieces as a 5×5 *square or as two squares, namely a* 3×3 *square and a* 4×4 *square.*

3. The two Holes





4. The Puzzle pieces

The unit length here is not specified. All right triangles are similar to the triangle with side-lengths 3,4,5.

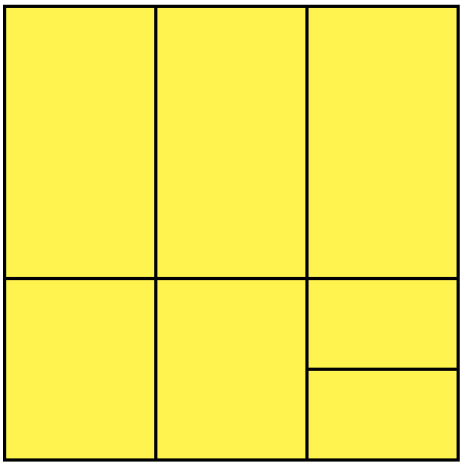
First puzzle: Seven yellow puzzle pieces form either a 15×15 square or a 9×25 rectangle. The yellow pieces are: three of size 9×5, two of size 6×5, two of size 3×5.

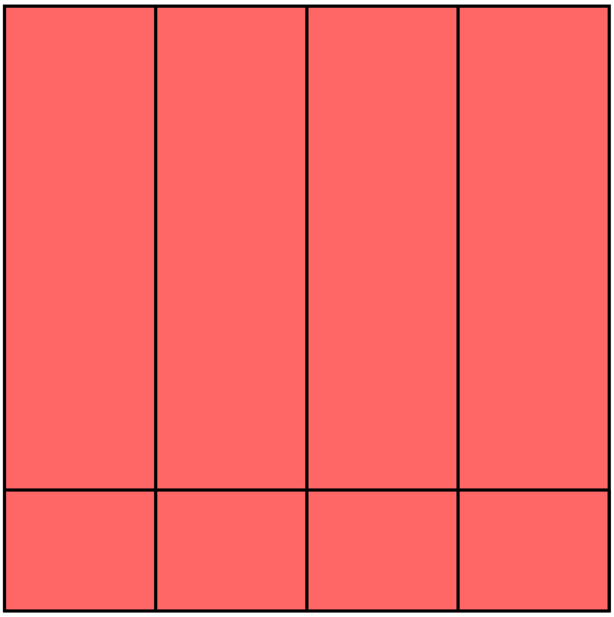
Second puzzle: Eight red puzzle pieces form either a 20×20 square or a 16×25 rectangle. The red pieces are: four pieces of size 16×5, four pieces of size 4×5.

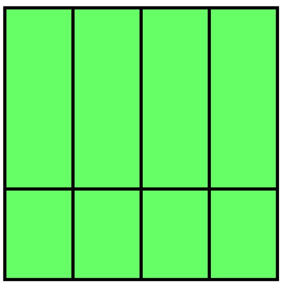
Third puzzle: Eight green puzzle pieces form either a 12×12 square or a 9×16 rectangle. The green pieces are: four of size 8×3, four of size 4×3.

Two special pieces: There are two special blue triangular pieces: together they form a right triangle with sizes $15, 20, 25$. The blue pieces are: One right triangle of side-lengths $9, 12, 15$ and one right triangle of side-lengths $12, 16, 20$.

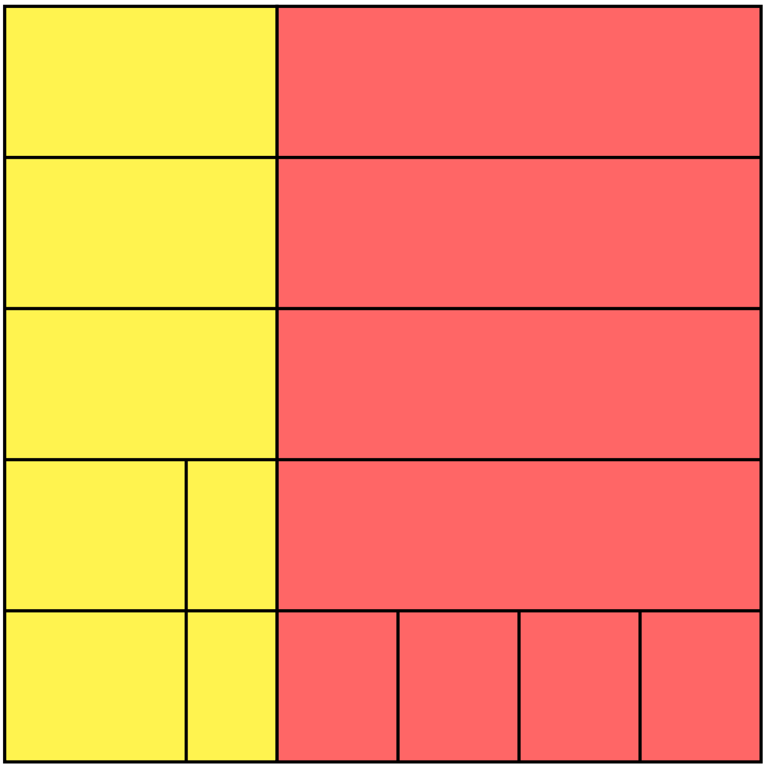
5. First configuration for the three puzzles

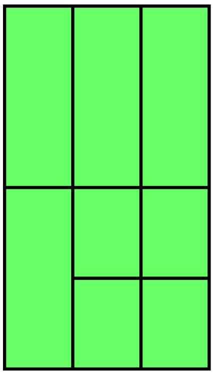






6. Second configuration for the three puzzles





7. Text panel

On the table there is some text, namely some explanation and some hidden solution. This can be provided in several languages.

The explanation text could be as follows:

Some results about right triangles are illustrated here.
What can they be?

The solution could be as follows:

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1. Pythagoras' Theorem: The square of the hypotenuse equals the sum of the squares of the two legs.
2. The square of a leg equals the product of the hypotenuse and the projection of the leg on the hypotenuse.
3. The square of the altitude (the altitude towards the hypotenuse) equals the product of the projections of the two legs on the hypotenuse.
4. The altitude towards the hypotenuse cuts a right triangle into two right triangles which are similar to it.

Math-Challenge: Can you prove these results? You can show 4. by comparing the angles. Then you can prove 2. and 3. with the proportionality between similar triangles. Pythagora's Theorem follows from applying 2. to the two legs.
