

MASTERMIND

	●	●	●	●
■				
■				
■				
■				
● ● ● ●	●	●	●	●
● ● ■	●	●	●	●
● ● ■	●	●	●	●
● ■ ■	●	●	●	●

The game “Who is it?”

There are 24 people who can be identified through some characteristic traits.

One player selects one person, the other player tries to identify it with Yes/No questions, for example:

- ▶ “Do they wear glasses?”
- ▶ “Do they wear glasses or do they have white hair?”

The best questions are such that the answer is Yes for half of the people and No for the other half. In this way one can be sure to discard many possibilities.

Mastermind with symbols

	●	●	●	●
■				
■				
■				
■				
● ● ● ●	●	●	●	●
● ●	●	●	●	●
● ●	●	●	●	●
●	●	●	●	●

	E	F	A	C
■				
■				
■				
■				
40	E	F	A	C
02	C	A	C	E
02	A	C	D	D
01	A	A	B	B

Mastermind

- ▶ The **codemaker** chooses a sequence of 4 letters among A,B,C,D,E,F. The **codebreaker** presents a test code.
- ▶ **Reply:**
One black peg for each correct color in the correct position.
One white peg for each correct color in an incorrect position.

ABBB	CAAA	01
	CDEF	00
	ABBB	40



Replies

00	01	02	03	04
10	11	12	13	
20	21	22		
30				
40 😊				

- ▶ The reply **31** can never occur!
- ▶ With the replies **04 13 22** one has only to rearrange the test code.
- ▶ For example **01** is not possible for the test code **AAAA**

Game situation: Think!

- ▶ One has enough information to guess the code:

AABB

04

- ▶ One has enough information to guess the code:

CDEF

02

CDAC

22

Game situation: 50/50?

- ▶ There are exactly **2** codes left. The best strategy is trying one of those. One wins right away or in the next move!

CCDE

04

Der Algorithmus von Knuth

- ▶ Donald Knuth developed with the computer a strategy to always win in ≤ 5 moves!
- ▶ The strategy tries codes up to one of the following situations:



Think!

50/50?

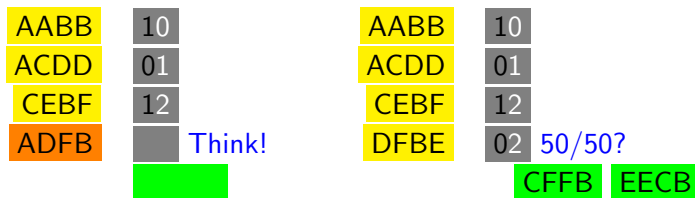
- ▶ **Idea:** Choose a test code such that every possible reply discards many codes!
- ▶ The first move is AABB.
Equally good would be for example FFDD or DFFD or FDFD.

Remaining codes after the first reply

	AAAA	AAAB	AABB	AABC	ABCD
00	625	256	256	81	16
01		308	256	276	152
02		61	96	222	312
03			16	44	136
04			1	2	9
10	500	317	256	182	108
11		156	208	230	252
12		27	36	84	132
13				4	8
20	150	123	114	105	96
21		24	32	40	48
22		3	4	5	6
30	20	20	20	20	20
40	1	1	1	1	1

Stroke of genius

Sometimes it is better to use a test code although one knows that it is not the right one.



After the first 3 moves there are 7 codes left:

DFBE FFBC FDBE EECB CFCB DEFB CFFB

None of these would always make us win in ≤ 5 moves!

Knuth's strategy...

Figure 1

1296(1122: 1,16(1213: 0,0,0,0,0;1,4(1415),3(1145),0;1,3(4115),3(1145);0,1;0),96A,256B,256C;0,36D,208E,256F;
4(1213),32G,114H;0,20I;1) ,

where

A = (2344: 0,2,16(3215: 0,0,0,0,0;1,2,1,1;2,3(3231),2;0,3(3213);1),
14(5215: 0,0,0,0,0;0,1,3(3511),3(3611);1,1,2;0,2;1),4(1515);0,6(2413),
18(2415: 1,1,0,0,0;1,2,3(2253),3(2236);1,2,2;0,1;1),15(2256x);0,4(2234),14(3315x);0,3(2314);0)

B = (2344: 0,7(2335),41(3235: 0,0,2,3(4613),2;0,3(5263),6(3413),6(3416);2,4(3256),6(1336);0,6(1536);1),
44(3516: 1,4(4651),6(6255),1,0;3(5613),7(1461),5(4551),1;3(1113),5(3551),3(4515);0,4(1145);1),
16(5515: 0,0,1,1,0;0,2,2,1;1,1,3(1516);0,3(1516);1);
2,21(3245: 1,3(2436),0,0,0;2,2,2,0;2,3(3234),2;0,3(3243);1),
42(4544: 1,1,7(2456),4(2635),3(2636);0,4(1356),5(4361),6(1635);2,2,3(3614);0,3(4414);1),
34(3315: 0,0,3(5641),4(2566),1;1,4(5361),4(5614),5(6614);2,4(3331),1;0,4(3316);1);
3(2434),13(2425x),23(1545: 0,1,3(2654),3(2353),4(1136);0,2,4(2564),3(2335);0,0,2;0,1;0);
0,9(1335x);1)

C = (3345: 2,20(4653: 2,2,0,0,0;3(4536),3(4534),1,0;2,2,1;0,3(4453);1),
42(6634: 0,3(4566),4(4556),1,0;2,5(4656),6(5653),4(1444);2,5(5636),5(4654);0,4(1413);1),
16(6646: 0,0,1,0,0;0,3(1416),1,1;3(1416),3(5666),2;0,2;0),1;
4(3453),40(3454: 1,5(4535),6(1436),0,0;2,5(4356),6(3536),0;1,3(3564),6(3463);0,4(3456);1),
46(3636: 1,1,3(4364),6(4565),6(4544);0,5(4366),6(1565),6(4546);2,4(3466),3(3556);0,2;1),
18(3656: 0,1,1,1,1;0,3(5665),3(6446),3(4446);0,1,3(4646);0,1;0);
5(3435x),20(3443: 0,0,4(4355),0,0;0,3(3334),4(3356),0;1,2,4(3455);0,1;1),
29(3636: 0,1,3(5365),4(6445),4(1444);0,2,3(3565),4(4645);1,1,4(3446);0,2;0);0,12(3446x);1)

■ ■ ■

D = (1213: 1, 4(1145), 3(1415), 0, 0; 0, 6(1114x), 7(2412x), 0; 2, 4(1145), 4(1145x); 0, 4(1114x); 1)

E = (1134: 0, 4(1312), 24(3521: 1, 2, 4(4612), 0, 0; 0, 3(3312), 3(2423), 0; 2, 2, 3(4621); 0, 3(3321); 1),
 38(2352: 2, 4(3226), 4(5621), 1, 0; 1, 5(2223), 7(6242), 1; 2, 4(2323), 4(2462); 0, 2; 1),
 20(2525: 1, 2, 1, 0, 0; 0, 3(2252), 3(2262), 0; 2, 2, 2; 0, 3(2225); 1);
 4(1341), 34(1315: 1, 3(4151), 4(4161), 0, 0; 1, 6(6451), 6(1461), 0; 3(1351), 3(1361), 2; 0, 4(1113); 1),
 32(1516: 2, 2, 3(2145), 0, 4(2324); 2, 4(1661), 4(1245), 0; 3(1561), 3(1551), 1; 0, 3(1511); 1),
 22(1256: 1, 0, 4(2524), 2, 0; 0, 2, 4(5224), 4(2224); 2, 0, 0; 0, 2; 1); 4(1314), 12(1315x), 12(1235x); 0, 2; 0)

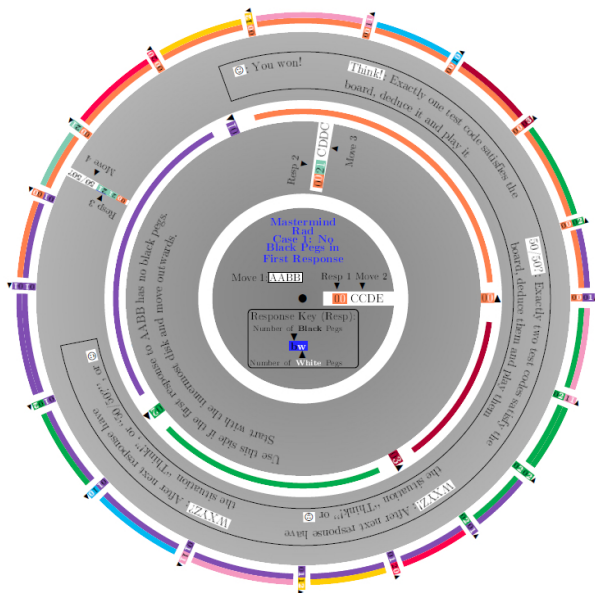
F = (1344: 0, 7(1335), 41(3135: 0, 0, 2, 3(4623), 2; 0, 3(5163), 6(3423), 6(3426); 2, 4(3156), 6(1436); 0, 6(1536); 1),
 44(3526: 1, 4(4652), 6(6155), 1, 0; 3(5623), 7(1462), 5(4552), 1; 3(1123), 5(3552), 3(4525); 0, 4(1145); 1),
 16(5525: 0, 0, 1, 1, 0; 0, 2, 2, 1; 1, 1, 3(1516); 0, 3(1516); 1);
 2, 21(3145: 1, 3(1436), 0, 0, 0; 2, 2, 0; 2, 3(3134), 2; 0, 3(3143); 1),
 42(4524: 1, 1, 7(1456), 4(1635), 3(1636); 0, 4(1356), 5(4362), 6(1336); 2, 2, 3(3624); 0, 3(4424); 1),
 34(3325: 0, 0, 3(5642), 4(1566), 1; 1, 4(5362), 4(5624), 5(6624); 2, 4(3332), 1; 0, 4(3326); 1);
 3(1434), 13(1415x), 23(1415: 0, 0, 2, 4(3324), 0; 0, 4(1546), 4(1356), 4(1136); 0, 2, 3(1136); 0, 0; 0); 0, 9(1335x); 1)

G = (1223: 1, 4(2145), 3(4115), 0, 0; 0, 5(2145), 6(4512), 0; 2, 4(1245), 3(1415); 0, 3(1145); 1)

H = (1234: 2, 16(1325: 1, 3(4152), 3(4162), 0, 0; 1, 3(3126), 2, 0; 1, 1, 1; 0, 0; 0),
 20(1325: 0, 3(5162), 1, 0, 0; 0, 2, 4(4522), 4(4622); 0, 3(5125), 3(2116); 0, 0; 0),
 6(2515), 0; 4(1323), 21(1352: 0, 1, 2, 0, 0; 2, 4(1623), 2, 0; 1, 3(1323), 3(1462); 0, 2; 1),
 16(2156x), 12(1315x); 2, 6(3526), 8(1536x); 0, 1; 0)

I = (1223: 0, 0, 0, 0, 0; 1, 5(1145x), 4(1114x), 0; 1, 3(1415), 4(1114x); 0, 2; 0)

...on Lang's Mastermind Rad



Variants of Mastermind

- ▶ Change the length of the code or the amount of colors
- ▶ Do not allow repetitions of colors

- ▶ **Statistical Mastermind**

With these seven test codes one can guess any code:

ABBA

BCED

CCAA

DEBD

EFEF

FFDC

References

- ▶ Donald E. Knuth. *The computer as master mind*. Journal of Recreational Mathematics, 9(1):1–6, 1976/77.
- ▶ Alexander Lang. *Mastermind*. Zulassungsarbeit Lehramt Realschule (2016), University of Regensburg. Supervised by Antonella Perucca.
- ▶ Eric W. Weisstein. *Mastermind*. From MathWorld—A Wolfram Web Resource. <http://mathworld.wolfram.com/Mastermind.html>